Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2010

APPLIED MATHEMATICS – ORDINARY LEVEL

FRIDAY, 25 JUNE – MORNING 9.30 to 12.00

Six questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be 10 ms^{-2}.

$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

Marks may be lost if necessary work is not clearly shown.

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1. A car travels along a straight level road.
It passes a point P at a speed of $12\ \text{ms}^{-1}$ and accelerates uniformly for 6 seconds to a speed of $30\ \text{ms}^{-1}$.
It then travels at a constant speed of $30\ \text{ms}^{-1}$ for 15 seconds.
Finally the car decelerates uniformly from $30\ \text{ms}^{-1}$ to rest at a point Q.
The car travels 45 metres while decelerating.

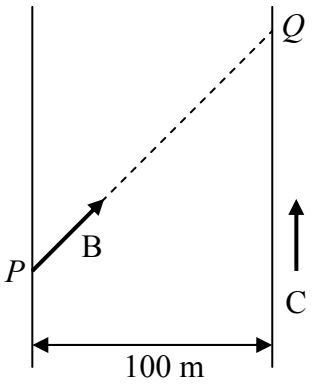
Find **(i)** the acceleration
(ii) the deceleration
(iii) $|PQ|$, the distance from P to Q
(iv) the average speed of the car as it travels from P to Q.

2. A river is 100 metres wide and has parallel banks.

Boat B departs from point P on its western bank and lands at point Q on its eastern bank.

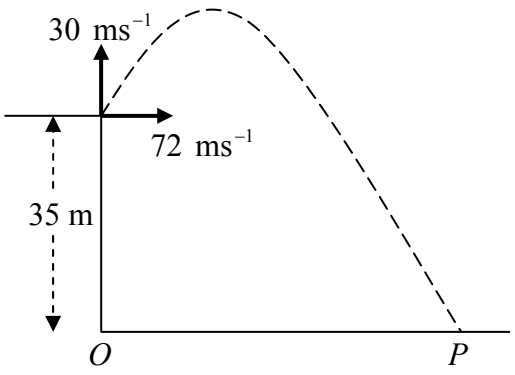
The actual velocity of the boat is $5\,\vec{i} + 12\,\vec{j}\ \text{ms}^{-1}$.

Cyclist C travels due north at a constant speed of $3\ \text{ms}^{-1}$ along the eastern bank of the river.



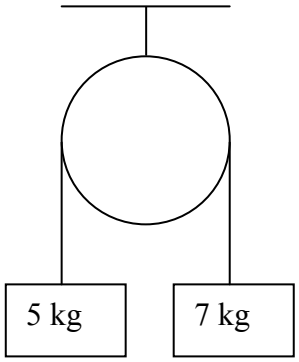
Find **(i)** the velocity of C in terms of $\vec{i}$ and $\vec{j}$
(ii) the velocity of B relative to C in terms of $\vec{i}$ and $\vec{j}$
(iii) the magnitude and direction of the velocity of B relative to C
(iv) the time it takes B to cross the river
(v) $|PQ|$, the distance from P to Q.

3. A particle is projected with initial velocity $72\,\vec{i} + 30\,\vec{j}\ \text{ms}^{-1}$ from the top of a straight vertical cliff of height 35 m.
It strikes the horizontal ground at P.



Find
(i) the time taken to reach the maximum height
(ii) the maximum height of the particle above ground level
(iii) the time of flight
(iv) $|OP|$, the distance from O to P
(v) the speed of the particle as it strikes the ground.

4. **(a)** Two particles of masses 5 kg and 7 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley.

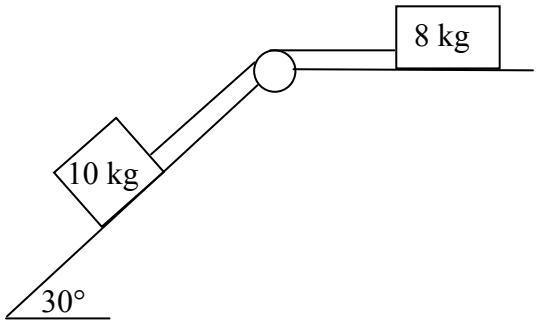


The system is released from rest.

Find **(i)** the common acceleration of the particles

(ii) the tension in the string.

(b) Masses of 8 kg and 10 kg are connected by a taut, light, inextensible string which passes over a smooth light pulley as shown in the diagram.



The 8 kg mass lies on a rough horizontal plane and the coefficient of friction between the 8 kg mass and the plane is $\frac{1}{2}$.

The 10 kg mass lies on a smooth plane which is inclined at 30° to the horizontal.

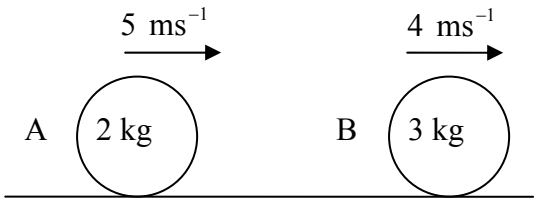
The system is released from rest.

(i) Show on separate diagrams the forces acting on each particle.

(ii) Find the common acceleration of the masses.

(iii) Find the tension in the string.

5. A smooth sphere A, of mass 2 kg, collides directly with another smooth sphere B, of mass 3 kg, on a smooth horizontal table.



A and B are moving in the same direction with speeds of $5\ \text{ms}^{-1}$ and $4\ \text{ms}^{-1}$ respectively.

The coefficient of restitution for the collision is $\frac{2}{3}$.

Find **(i)** the speed of A and the speed of B after the collision

(ii) the change in the kinetic energy of A due to the collision

(iii) the magnitude of the impulse imparted to A due to the collision.

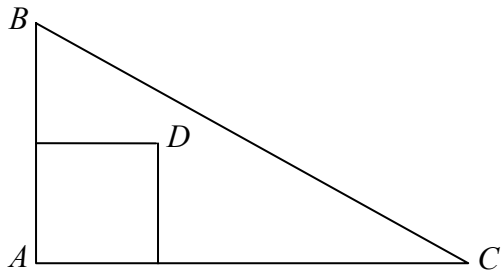
6. **(a)** Particles of weight 3 N, 7 N, 1 N and 5 N are placed at the points $(1, p)$, $(p, 2)$, $(-2, q)$ and $(4, 8)$, respectively.
The co-ordinates of the centre of gravity of the system are $(3 \cdot 5, 4 \cdot 5)$.

Find **(i)** the value of p

(ii) the value of q.

(b) A triangular lamina with vertices A, B and C has the square portion with diagonal $[AD]$ removed.

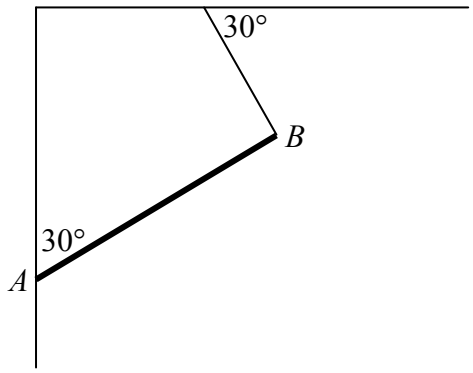
The co-ordinates of the points are $A(0,0)$, $B(0,9)$, $C(12,0)$ and $D(4,4)$.



Find the co-ordinates of the centre of gravity of the remaining lamina.

7. A uniform rod, $[AB]$, of length 2 m and weight 40 N is smoothly hinged at end A to a vertical wall.

One end of a light inelastic string is attached to B and the other end of the string is attached to a horizontal ceiling.



The string makes an angle of $30°$ with the ceiling and the rod makes an angle of $30°$ with the wall, as shown in the diagram.

The rod is in equilibrium.

(i) Show on a diagram all the forces acting on the rod $[AB]$.

(ii) Write down the two equations that arise from resolving the forces horizontally and vertically.

(iii) Write down the equation that arises from taking moments about point A.

(iv) Find the tension in the string.

(v) Find the magnitude of the reaction at the hinge, A.

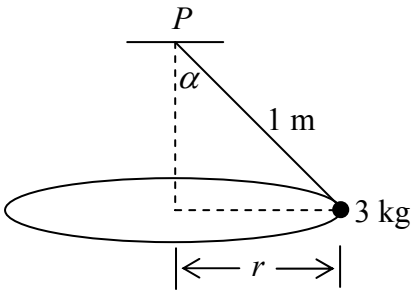
8. **(a)** A particle describes a horizontal circle of radius r metres with uniform angular velocity ω radians per second.
Its speed and acceleration are 6 ms^{-1} and 12 ms^{-2} respectively.

Find **(i)** the value of r
(ii) the value of ω .

(b) A conical pendulum consists of a particle of mass 3 kg attached by a light inelastic string of length 1 metre to a fixed point P.

The particle describes a horizontal circle of radius r. The centre of the circle is vertically below P.

The string makes an angle of α with the vertical where $\tan\alpha = \frac{4}{3}$.



Find **(i)** the value of r
(ii) the tension in the string
(iii) the angular velocity of the particle.

9. **(a)** State the Principle of Archimedes.

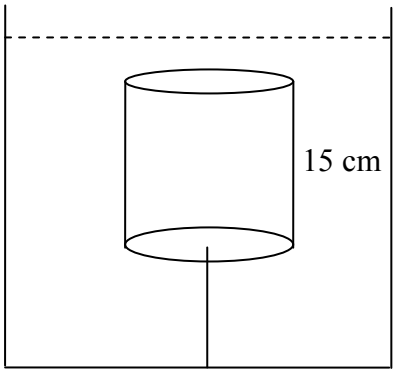
A solid piece of metal has a weight of 14 N.
When it is completely immersed in water the metal weighs 9 N.

Find **(i)** the volume of the metal
(ii) the relative density of the metal.

(b) A right circular solid cylinder has a base of radius 6 cm and a height of 15 cm.

The relative density of the cylinder is 0·7 and it is completely immersed in a tank of liquid of relative density 0·9.

The cylinder is held at rest by a light inextensible vertical string which is attached to the base of the tank. The upper surface of the cylinder is horizontal.



Find the tension in the string.

[Density of water = 1000 $kg\,m^{-3}$]

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